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Abstract

Equivalence scales provide answers to questions like how much a household with four children needs to spend compared to a household with two children or how much a childless couple needs to spend compared to a single person household to attain the same welfare level. These are important questions for child allowances, social benefits and to assess the cost of children over the life-cycle for example. The latter is also interesting from a theoretical point of view, especially if future events are allowed to be uncertain. We discuss equivalence scales in an intertemporal setting with uncertainty. To estimate equivalence scales we use subjective data on satisfaction with life and satisfaction with income to represent the welfare level. Because satisfaction is measured on a discrete scale we use limited dependent variable models in estimation. The results are based on a panel from German households (GSOEP). Using satisfaction with life data we find that larger households do not need any additional income to be as satisfied with their life as a couple. Using satisfaction with income, however, indicates that an increase in the household size leads to a significant drop in the satisfaction with their income. This result is used to compute equivalence scales.

Keywords: (lifetime) equivalence scales, panel data, parametric models

JEL classification: C23, C25, I31

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1. Introduction

Equivalence scales indicate how much expenditure a household with a given demographic composition needs to reach the same welfare level as a reference household with a different demographic composition. For example, equivalence scales provide answers to questions like how much expenditure a household with four children needs compared to a household with two children, or how much expenditure a childless couple needs compared to a single person household, to attain the same welfare level. The answers to these questions are important, because, for example, poverty thresholds, child allowances and social benefits are based on them. In an intertemporal setting they can be used to assess the cost of children over the life-cycle. However, computation of equivalence scales is not obvious when future events, like income for example, are uncertain, which makes equivalence scales in an intertemporal setting also interesting from a theoretical point of view. Furthermore, the translation of equivalence scales to economic models requires explanation of what is meant by "expenditure" and "welfare level". Estimation of equivalence scales also requires data.[2] Using cross-section models, several approaches are available in the literature. In all these models expenditure in the definition of equivalence scales is replaced by income. We will discuss three approaches.

The first approach relies on demand systems. The term "welfare level" in the definition of equivalence scales is replaced by "utility" and equivalence scales are just ratios of the expenditure of attaining a given utility level for households with different demographic characteristics. The utility function in the model determines demand equations that can be estimated. However, these demand equations do not fully identify cost functions and, therefore, equivalence scales are not identified on the basis of demand data alone. This is shown by Pollak and Wales (1979) and Blundell and Lewbel (1991) discuss the identification issue in more depth.

The second approach also relies on micro-economic models and again the term "welfare level" in the definition of equivalence scales is replaced by "utility". However, instead of using indirect measures of the expenditure function, data that reveal the expenditure function directly are used in estimating equivalence scales. To construct the expenditure function a question concerning the level of income the household would consider ‘very bad’, ‘bad’, ‘insufficient’, ‘sufficient’, ‘good’ or ‘very good’ in their current circumstances is used. This is a so called Income Evaluation Question (IEQ) used by, for example, Van Praag and Kapteyn (1973), Van Praag and Van der Sar (1988), and Melenberg and Van Soest (1996). It can be interpreted as a direct measure of the expenditure needed to attain a given utility level. Equivalence scales can then be estimated from

[2] Equivalence scales used in policy are not only based on econometric models but they are also expert-based.

these estimated expenditure functions.

The third approach does not require an explicit micro-economic model. Instead, subjective data on satisfaction are used and hence the term "welfare level" in the definition of equivalence scales is replaced by "satisfaction level". The relationship between satisfaction and income and demographic characteristics of the household is specified and the parameters in this relationship are estimated using satisfaction data and an ordered response model. Melenberg and Van Soest (1996) find that the results based on satisfaction data lead to much more plausible results than the results based on the IEQ. They state that a reason for this finding might be that the IEQ asks for information in some virtual situations, whereas the satisfaction question refers to the household’s actual situation. In this paper we will therefore use satisfaction data to estimate equivalence scales for Germany. For an overview of studies using subjective data (mainly IEQ data) see Kapteyn and Wansbeek (1985).

In our application we will use the German Socio-Economic Panel (GSOEP). Satisfaction data for Germany, based on the GSOEP, have been analyzed in a descriptive fashion in sociology. For example, using the 1984 through 1992 waves, Andreβ (1996) investigates satisfaction with income as well as changes in satisfaction with income. He concludes that the main explanatory variables are income, labour market state and demographics (like civil status or household size). Using the 1984 through 1987 waves, Landau (1992) concludes that changes in satisfaction with life can be traced back to changes in living conditions like employment, health and family composition. For the satisfaction level also the sex of the head of the household matters. An econometric model instead of just a descriptive analysis was used by Winkelmann and Winkelmann (1995). They use satisfaction with life data from 1984 through 1989 to examine the social cost of unemployment. Satisfaction below a given level is defined as zero and the other values are coded as one. A binary choice panel data model with fixed or random effects is estimated based on a balanced panel.

One of the problems with subjective data is that two households that have the same welfare level might answer the questions on satisfaction differently. This problem can be accommodated using panel data and allowing for household specific effects that can be correlated with the regressors. Adding the household specific effects allows households that are exactly the same in their observable characteristics to differ in average satisfaction level and hence average welfare level, where the average is taken over time. Because satisfaction is measured on a discrete scale we will use ordered response panel data models (ORPD) in estimating equivalence scales.

However, in an intertemporal context, expenditure in a given period need not be equal to income in a given period (intertemporal substitution). Furthermore, the definition of welfare depends on the period that is considered. Equivalence scales based on a specific period will be referred to as period-specific equivalence scales. Equivalence scales based on lifetime welfare will be referred to as lifetime equivalence scales. In an intertemporal setting with uncertainty, the definition of lifetime welfare is not obvious and needs some clarification, so lifetime equivalence scales are interesting from a theoretical point of view. Furthermore, period-specific equivalence scales can differ for each period of the life-cycle and they can depend on the age of children, for example. Therefore, comparing two households that differ in terms of composition during (part of) the lifetime, would lead to a sequence of period-specific equivalence scales. A lifetime equivalence scale is just a single number, which is a neat way to present the results from the comparison.

In section 2 we discuss the notion of equivalence scales in an intertemporal setting, in section 3 we describe the data, in section 4 we explain the model we use in estimation and in section 5 we present estimation results. Section 6 concludes.

2. Equivalence scales in an intertemporal setting

The definition of equivalence scales contains the terms "expenditure" and "welfare level". In micro-economic cross-section models, expenditure is usually set equal to income. In an intertemporal setting expenditure in a given period need not be equal to income in that period due to the possibility of intertemporal substitution. Furthermore, future income, for example, might be uncertain. Whether the future is certain or uncertain influences the definition of "expenditure" and "welfare". In this section we will discuss five approaches to estimate equivalence scales in an intertemporal setting. These will be referred to as lifetime equivalence scales. In all approaches we will explain what is meant by "welfare", which intertemporal micro-economic model is used (if any) and what type of data is used to estimate lifetime equivalence scales. An overview on how these approaches relate in terms of the model and data used is given below. Comparing the definitions of "welfare" is rather involved, so for a discussion, see the detailed explanations of the five approaches below.

Approach	model	data used
1	life-cycle with certainty	demand data
2	life-cycle with uncertainty	demand data
3	life-cycle with uncertainty	satisfaction data
4	based on interpretation of data	satisfaction data
5	based on interpretation of data	satisfaction data

The first three approaches use a life-cycle model with a utility function that is assumed to be intertemporally additive. To describe these approaches in more detail we need some notation:

x_t is total expenditure in period t,

p_t is a vector of period t commodity prices,

h_t is household composition in period t,

u_t is discounted utility in period t,

w_t is the income in period t,

A_t is the value of assets at the beginning of period t,

r_t is the real interest rate in period t

I_t is the information on period t variables, $I_t=\{w_t, p_t, r_t, h_t\}$,

C is the end of the lifetime,

h is $(h_0,..,h_C)$,

$\mathscr{F}_t$ is all the information available at the beginning of period t, satisfying $\mathscr{F}_1\subseteq\mathscr{F}_2\subseteq...\subseteq\mathscr{F}_C$,

E_t denotes expectation over $\mathscr{F}_C$ conditional on $\mathscr{F}_t$.

To compute equivalence scales we will compare a reference household (consisting of two adults) with a comparison household. Therefore we will introduce some additional notation. Let I^0_t denote the information related to a specific period t for a reference household, $I^0_t=\{w^0_t, p_t, r_t, h^0_t\}$ and let I^1_t denote the information related to a specific period t for a comparison household, $I^1_t=\{w^1_t, p_t, r_t, h^1_t\}$. Hence they only differ in terms of w_t and h_t. All the available information at the beginning of period t is denoted by $\mathscr{F}^0_t$ and $\mathscr{F}^1_t$, respectively. Let E^j_t denote expectation conditional on $\mathscr{F}^j_t$, j=0,1. What is in $\mathscr{F}_t$ depends on whether uncertainty is allowed for or not. Under certainty $\mathscr{F}_t=\mathscr{F}_C=I_0\cup...\cup I_C$ whereas under uncertainty $\mathscr{F}_t=I_0\cup I_1\cup...\cup I_t$. Similar expression hold for $\mathscr{F}^0_t$ and $\mathscr{F}^1_t$.

We will restrict attention to a life-cycle model in which lifetime utility U is intertemporally additive in (discounted) within-period utility u_t and in which preferences over household composition are intertemporally additive as well. So

$$U = \sum_{t=0}^{C} \{ u_t(x_t,p_t,h_t) \ + \ f_t(h_t)\}$$

where $\Sigma^C_{t=0}f_t(h_t)$ reflects preferences over household composition over the life-cycle.

At the beginning of period t, a household maximizes expected utility as of period t subject to a budget constraint, i.e.

$$\max_{x_t,..,x_C} \ E_t\left\{\sum_{\tau=t}^{C} \{ u_\tau(x_\tau,p_\tau,h_\tau) + f_\tau(h_\tau)\} \right\} \tag{1}$$
$$\text{s.t. } A_{\tau+1} \ = \ (1+r_\tau)(A_\tau+w_\tau-x_\tau), \ \tau\geq t, \ A_{C+1}=0$$

The <u>first approach</u> we discuss in detail is presented in Banks et al. (1994a). In the absence of uncertainty, the expectation operator in maximization problem (1) can be dropped. Furthermore, Banks et al. (1994a) replace u_t by $F_t(v_t(x_t,p_t,h_t))$, where $v_t(x_t,p_t,h_t)$ is an indirect utility function representing within-period preferences. In estimating equivalence scales Banks et al. (1994a) proceed in three steps. In the first step the parameters in $v_t(x_t,p_t,h_t)$ are estimated using the UK Family Expenditure Survey (FES) from 1969 through 1988. To explain the second step, let λ_t denote the marginal utility of expenditure, i.e. $\lambda_t=\partial F_t/\partial x_t$. The optimal expenditure path follows from the equation $\lambda_t=\lambda_{t+1}$, see Deaton (1992) for example. Using observations on x_t from the FES this equation can be used to estimate the parameters in F_t. All the parameter estimates are used to construct the optimal expenditure path for a reference household and the resulting optimal expenditure path yields the optimal value for $\Sigma_{t=0}^{C}u_t(x_t,p_t,h_t^0)$, i.e. lifetime utility U excluding $\Sigma_{t=0}^{C}f_t(h_t^0)$. In the third step, we construct equivalence scales. Therefore we need optimal expenditure paths to reach the same lifetime utility as a comparison household. Because demand data do not identify $\Sigma_{t=0}^{C}f_t(h_t)$, lifetime utility is not identified and hence equivalence scales will depend on the choice for $f_t(h_t)$. Banks et al. (1994a) are very much aware of this and they start assuming $\Sigma_{t=0}^{C}f_t(h_t)=0$. With this specification for U, their results for lifetime equivalence scales are "too high when judged against what seems intuitively reasonable".[3] This is stated in Banks et al. (1994b) who report the same lifetime equivalence scales as Banks et al. (1994a). To bring the lifetime equivalence scales at a more plausible level they choose a specific $\Sigma_{t=0}^{C}f_t(h_t)$ such that lifetime equivalence scales are intuitively more reasonable.

For a notion of equivalence scales it is important to distinguish certainty from uncertainty. In <u>approach two</u> we extend the Banks et al. (1994a) approach to a life-cycle model with uncertainty. This model will still suffer from the same identification problem as the previous model but emphasis here is on the definition of equivalence scales. The same definition of equivalence scales will be used in a model using different type of data that identify $\Sigma_{t=0}^{C}f_t(h_t)$. In a life-cycle model with uncertainty the information available at the beginning of period t is modelled to be

$$\mathscr{F}_t = I_0 \cup I_1 \cup \ldots \cup I_t = \{w_0,..,w_t,\ p_0,..,p_t,\ r_0,..,r_t,\ h_0,..,h_t\}$$

Compared to the approach in Banks et al. (1994a), step one of the estimation procedure does not change due to the intertemporal additivity of lifetime utility. To determine optimal expenditure levels under uncertainty, the relation $\lambda_t=\lambda_{t+1}$ in step two is replaced by $\lambda_t=E_t\lambda_{t+1}$ (see Deaton, 1992, for example) and this is used to estimate the parameters in F_t. However, step three, the construction of equivalence scales is more complicated due to uncertainty. Under uncertainty, an obvious definition of lifetime welfare is expected lifetime utility rather than lifetime utility. Furthermore, *ex ante* optimal expenditure levels are random variables because they depend on future optimal expenditure levels which, in turn, will depend on the particular realizations for the uncertain

[3] Banks et al. (1994a) use r_t=0.05 but they do not explain how they choose $p_0,..,p_C$ and $w_0,..,w_C$.

variables in these future periods, like income or prices, for example. Therefore lifetime equivalence scales will be random variables.[4] However, *ex post* equivalence scales may be estimated.[5] At the beginning of period t, a reference household uses its information, $\mathscr{F}_t^0$ say, and its budget constraint to determine the optimal expenditure level x_t^*. This gives a value for the expected utility as of period t, $E_t^0\Sigma_{\tau=t}^C\{u_\tau(x_\tau^*,p_\tau,h_\tau)+f_\tau(h_\tau)\}$ for the reference household. A household with a different household composition in period t for which $E_t^1\Sigma_{\tau=t}^C\{u_\tau(x_\tau,p_\tau,h_\tau)+f_\tau(h_\tau)\}$ equals $E_t^0\Sigma_{\tau=t}^C\{u_\tau(x_\tau^*,p_\tau,h_\tau)+f_\tau(h_\tau)\}$ will choose a different optimal value for x_t, when compared to the reference household. Comparing the optimal values for x_t yields period-specific equivalence scales, e_t say, based on the same expected utility as of period t. These e_t depend upon all information up to period t for both a reference household and a comparison household, i.e. $e_t=e_t(\mathscr{F}_t^0,\mathscr{F}_t^1)$. To compute lifetime equivalence scales the e_t have to be estimated for each period of the life-cycle.[6] Given these period-specific equivalence scales, lifetime equivalence scales, discounted by a time-constant nominal interest rate r, can be computed as

$$\frac{\sum_{t=0}^{C}(1+r)^{-t}x_{1t}}{\sum_{t=0}^{C}(1+r)^{-t}x_{0t}} = \frac{\sum_{t=0}^{C}(1+r)^{-t}e_t(\mathscr{F}_t^0,\mathscr{F}_t^1)\,x_{0t}}{\sum_{t=0}^{C}(1+r)^{-t}x_{0t}}$$

To estimate lifetime equivalence scales we need a pattern for x_{0t}. For simplicity we will use the expenditure pattern resulting from a life-cycle model with certainty and intertemporally additive lifetime utility over quantities of individual goods in which the rate of time preference is equal to the real interest rate. Then, the optimal x_t divided by a price index are constant over time, see Deaton (1992), for example. If we let the price index increase at rate ρ, then x_{0t} grows at rate ρ and $x_{0t}=(1+\rho)^t x_{00}$. It is easy to show that the lifetime equivalence scale can be approximated by

[4] The exception to this rule is a model with dynamically complete markets. Then, by definition, any future (random) expenditure stream can be obtained with enough additional initial funds and appropriate investments in the various securities. Hence, optimal future expenditure streams then do not depend on realizations of uncertain variables in these future periods.

[5] In the remainder we will drop the *ex post* prefix in period-specific and lifetime equivalence scales.

[6] However, using demand data does not identify $f_t(h_t)$ so the same identification problem appears.

$$\frac{\sum_{t=0}^{C} e_t(\mathscr{F}^0_t,\mathscr{F}^1_t) \; + \; (\rho-r)\sum_{t=0}^{C} t\, e_t(\mathscr{F}^0_t,\mathscr{F}^1_t)}{C \; + \; (\rho-r)C(C+1)/2} \qquad \textbf{(2)}$$

ignoring terms of order $(\rho-r)^2$ and higher.

If we assume that the price index increases at rate r (i.e. ρ=r) and hence keeping real expenditure of the reference household constant in all periods, the lifetime equivalence scale is just the average of the period-specific equivalence scales. Lifetime equivalence scales will be smaller than the period-specific equivalence scales because the period-specific equivalence scales for periods where no children are present are equal to one and these ones will bring down the (weighted) average over the lifetime.

Approaches one and two use indirect measurement of lifetime utility U by means of expenditure data leading to a similar identification problem as in the cross-section model using demand data: the function $\Sigma_{t=0}^{C} f_t(h_t)$ can not be identified using demand data and hence lifetime equivalence scales still cannot be identified.

As in the cross-section model, the identification issue can be solved using other type of data. Direct measurement of expenditure functions using the IEQ question as described in the second approach in the introduction could be a solution. To the best of our knowledge, applications of this approach using panel data are not available. Because Melenberg and Van Soest (1996) find that the results based on satisfaction data lead to more plausible results than the results based on the IEQ, we will focus on extending the third approach in the introduction using panel data on satisfaction.

Approaches three through five discussed below rely heavily on the definition of period-specific and lifetime equivalence scales given before and on interpretations of the answers to the satisfaction question. We will give three possible interpretations that lead to the same equivalence scales. In approach three we use a life-cycle model under uncertainty as described above and we use a direct measure of welfare by means of satisfaction data. An interpretation of the answer to the satisfaction question is that it is the maximum expected utility as of period t, i.e.

$$E_t\left\{ \sum_{\tau=t}^{C} \{ u_\tau(x^*_\tau,p_\tau,h_\tau) + f_\tau(h_\tau) \} \right\}$$

or some monotonically increasing transformation of it. To be able to use the answers to the satisfaction question we have to relate them to explanatory variables. We will assume that the information in $\mathscr{F}_t$ can be summarized by I_t so the maximum expected utility as of period t and hence satisfaction in period t depends only on the past through I_t. The parameters in this relation can be estimated using an ORPD model, explained in section 4. Given the parameter estimates, period-specific equivalence scales can then be computed by equating satisfaction in period t for a comparison household with satisfaction for the reference household. These period-specific equivalence scales are then used to construct lifetime equivalence scales as described above. So we have given an example of a life-cycle model, that, together with an interpretation of the satisfaction data, can be used to compute equivalence scales. In this life-cycle model the period-specific equivalence scales can be interpreted as period-specific life-cycle consistent equivalence scales.

The interpretation of the satisfaction data used in approach three is not the only interpretation possible. In approach four an alternative interpretation is used. For instance, we assume that the head of household, when answering a satisfaction question in a given period t, performs a thought experiment in which future income, prices and the interest rate are kept at their level in period t. The head of household then answers how satisfied he will be as of period t. For a household with a given composition in period t, a definition of period-specific equivalence scales is the amount of money that is needed to attain the same satisfaction level in period t as the reference household. We also assume that satisfaction in period t only depends on period-t information. In the interpretation used here, this assumption is satisfied for income, prices and interest rates. For household composition this implies that either future household composition is the same as in period t or that future household composition is uncertain but that the distribution of household composition as of period t is completely determined by household composition in period t. An ORPD model discussed in section 4 can be used to estimate the parameters in the relation between satisfaction and period-t information. These parameter estimates will determine period-specific equivalence scales, e_t say, and lifetime equivalence scales follow by taking a weighted average,

$$\frac{\sum_{t=0}^{C} e_t + (\rho - r)\sum_{t=0}^{C} t\, e_t}{C + (\rho - r)C(C+1)/2} \qquad \textbf{(3)}$$

where r is the (time-constant) real interest rate, ρ is the time-constant rate of expenditure growth of the reference household and terms of order $(\rho - r)^2$ and higher are ignored, compare (2). In this interpretation, satisfaction data can be analyzed without a life-cycle model. However, the two interpretations of satisfaction data discussed in approaches three and four lead to the same estimates of period-specific and lifetime equivalence scales.

Finally, in approach five we assume that the head of household answers how satisfied he is in period t, based on his period t information. Period-specific equivalence scales, e_t, are again defined in terms of reaching the same satisfaction level as the reference household, see approach four. Lifetime equivalence scales are again a weighted average of these period-specific equivalence scales, see (3). Similar to approach four, an ORPD model discussed in section 4 can be used to estimate the parameters in the relation between satisfaction and period-t information. These parameter estimates will determine the e_t from which the lifetime equivalence scales follow easily. In this approach satisfaction data are analyzed without reference to a life-cycle model.

Approaches four and five are very similar. The main difference is that uncertainty of future events, like future household size for example, is taken into account in approach four whereas it is not in approach five. Approach three also takes future uncertainty into account but it has the important advantage that period-specific and lifetime equivalence scales can be interpreted in a life-cycle model. Without reference to an economic model, interpretation of the equivalence scales in approach four and five is not as clear.

3. Data

We use the German Socio-Economic Panel (GSOEP). This panel dataset consists of data as of 1984 on Germans born in the former West Germany and foreigners living in the former West Germany. As of 1990 it also contains data on Germans born in the former East Germany. The sampling strategy is such that for each of the three subsamples no new households are added except households that stem from the first wave of the subsample. The topics covered are household composition, employment and professional mobility, earnings development, housing and living conditions, regional mobility, health, occupational and family biographies, and personal satisfaction. Topics that were surveyed in only one year include social security, education and training, allocation of time and savings, and assets held by the households.

In this paper we will focus on estimation of period-specific and lifetime equivalence scales for Germans born in the former West Germany using personal satisfaction data. For computational reasons we will only use the data from 1984-1991 (see below). Information on the data can be found in Wagner et al. (1993) as well as on the world wide web.[7] The first wave of the panel is important in the sense that only the persons in the households that were selected for an interview in 1984 are followed over time, so the only new persons or households that are added stem from the initial households. To avoid strong relationships between the households used in estimation we determine the head of household in 1984 (from a question in the survey) and this person is followed over time. Any other persons or households stemming from these initial households are left out of our sample.

For the purpose of estimating equivalence scales, satisfaction with life and satisfaction with income can be used. Both variables are measured on a discrete scale from 0 (very dissatisfied) to

[7] URL is http://www.diw-berlin.de/soep/e.faltblat.html

10 (very satisfied). The questions are presented in appendix A. Household income is an important explanatory variable in this context. The head of household is asked to report the household net monthly income. The exact question is presented in appendix A. In table 1 we present definitions of the variables used in this study. Real income is nominal income divided by the consumer price index, as reported in the statistical yearbooks of Statistics Germany. In table 2 we present an overview of the data. It contains averages and standard errors for each variable in each year. It shows that, on average, satisfaction with life does not show a pattern in the early years but it is increasing from 1988 onwards. In contrast, the average satisfaction with income is increasing over time and it is always below the average satisfaction with life. The logarithm of real household income is increasing over time, on average. The average household size is constant over time. The fraction of people in full-time employment decreases slightly as of 1989, whereas the fraction being part-time employed is stable over time. The fraction unemployed decreases over time and the fraction nonparticipants increases from 1988 onwards.[8] All these changes can be due to the changing age distribution in the sample. It should be remembered that we only use the heads of households that responded in 1984. They are followed over time. No new households are considered because this would lead to dependencies between the households in the sample and hence the independence assumption over the households in the sample, a standard assumption in panel data models, cannot be justified. This implies immediately that, over time, the age distribution shifts to the right. This shift can explain the increase in real income and the decrease in full-time employment as of 1988, the decrease in the fraction unemployed, the increase in the fraction not participating and the decrease in average satisfaction with health.

Figure 1 contains several graphs. The upper graphs contain nonparametric estimates for the relation between satisfaction with life and log(income) and the relation between satisfaction with income and log(income). Both are based on a quartic kernel using a rule of thumb bandwidth which is equal to $M^{-1/5}$sd(log(income)) where M is the number of observations in the pooled sample and sd(log(income)) is the standard error of the explanatory variable log(income). These graphs also contain 95% uniform confidence bands for the nonparametric estimates based on Härdle and Linton (1994). According to both satisfaction measures satisfaction increases with log(income). Satisfaction with income rises sharper with log(income) than satisfaction with life does. The lower graphs give satisfaction with life and satisfaction with income averages for each size of household present in the sample. The circles represent the averages whereas the crosses

[8] Persons are unemployed if they are looking for a job or if they are willing to start in a job immediately or within a year. Persons are nonparticipating if they indicate so or if they state that they are looking for a job but would not accept any offer within a year.

represent the boundaries of 95% (pointwise) confidence bands. There is no clear relation between household size and satisfaction with life or satisfaction with income. Comparing the two, however, shows that household size has a larger effect on satisfaction with income than on satisfaction with life. The former effect, if nonzero, is negative. It is needless to say that these results do not correct for other characteristics of the family. In particular, household size and income will be positively related. In section 5 we will estimate a model taking additional explanatory variables into account.

Under the assumptions in the previous section, the optimal expenditure level in period t only depends on income in period t and on initial assets in period t and not on previous realizations of income. However, initial assets in each period are not available in our dataset. Therefore the effect of initial assets is assumed to be captured by the household specific effect and the remainder is absorbed in the error term. This is not a serious drawback when compared to the existing cross-section equivalence scale literature, because in the latter expenditure is just assumed to be equal to income and not to income plus (part of) the value of assets.

4. Models

In estimating period-specific equivalence scales we use the following Ordered Response Panel Data (ORPD) model:

$$\begin{cases} y_{it}^{*} = \beta_t' x_{it} + \alpha_i + u_{it}, & i=1,..,N,\ t=1,..,T \\ y_{it} = j \ \text{if} \ \gamma_j < y_{it}^{*} \leq \gamma_{j+1}, & \gamma_0 = -\infty,\ \gamma_1 = 0,\ \gamma_R = \infty,\ \gamma_j \leq \gamma_{j+1} \end{cases} \tag{4}$$

Here R is the number of possible outcomes for y_{it} (R=11), N is the number of households in the sample, T is the number of time periods, α_i, i=1,..,N, are the household specific effects and u_{it} are the error terms, i=1,..,N, t=1,..,T. x_{it} will include time dummies δ_t, t=1,..,T. We also assume independence over the individuals, i=1,..,N. Note further that β is allowed to vary over time. As we will explain below, we can allow the thresholds γ_j to vary over the individuals for the fixed effects model. To be able to estimate the parameters $\beta=(\beta_1',..,\beta_T')'$ we have to add distributional assumptions on α_i and/or u_{it} to this model. The additional assumptions are specified below.

To compare the estimation results for different models we will use fixed and random effects models as well as a pooled model. We start with a fixed effects parametric model adding the assumption that the u_{it}, i=1,..,N, t=1,..,T are independent of α and $(x_1,..,x_T)$ and that they follow an iid standard logistic distribution. Estimation can then be performed in the following two steps (see Das and Van Soest, 1996). In the first step we construct dummy variables $\tilde{y}_{itg}$ as

$$\tilde{y}_{itg} = \begin{cases} 0 & \text{if } y_{it} \leq g \\ 1 & \text{otherwise} \end{cases}$$

Hence if y_{it} is larger than some threshold g, then $\tilde{y}_{itg}$ equals one whereas it is zero otherwise, g=0,..R−2. Transforming the y_{it} variables into dummy variables together with the logistic distributional assumptions allows us to use the estimator for the fixed effect binary choice model with logistically distributed error terms to get a consistent estimator. For this model an estimator is available, based on a conditional likelihood, see Chamberlain (1980) for example. Whereas the likelihood depends on the parameters α_i, the conditional likelihood does not and the estimates for β resulting from maximizing the conditional likelihood are consistent. Note however that not all the coefficients of the time dummies δ_t, t=1,..,T, are identified so the coefficient related to δ_1 is normalized at zero. For each choice of g we can employ a fixed effects binary choice logit model and maximize a conditional log-likelihood to obtain consistent estimates. If we assume independence between the dependent variable and a dummy r_{it} equal to one if household i was observed in period t, and zero otherwise, using an (un)balanced panel hardly affects estimation procedure available for panels without missing observations. The approach is the same as in Das and Van Soest (1996). In the absence of attrition and selection bias the conditional likelihood contribution for household i for a given g now is as follows:

$$P(\tilde{y}_{i1g},..,\tilde{y}_{iTg} \,|\, x_{i1},..,x_{iT}, \sum_{t=1}^{T} r_{it}\tilde{y}_{itg}, \beta) = \frac{1}{\sum_{d \in B_{ig}} \exp(\sum_{t=1}^{T} r_{it}(d_t - \tilde{y}_{itg})\beta_t' x_{it})}$$

where

$$B_{ig} = \left\{ (d_1,..,d_T) \,|\, d_t \in \{0,1\},\ d_t = 0 \text{ if } r_{it} = 0,\ \sum_{t=1}^{T} d_t = \sum_{t=1}^{T} r_{it}\tilde{y}_{itg} \right\}$$

Furthermore, it is easy to show that the conditional likelihood contributions do not depend on the thresholds γ_{ij} if the γ_{ij} are allowed to vary with i. Due to the transformation of y_{it} to the binary variable $\tilde{y}_{itg}$, the thresholds can be absorbed into the fixed effect as long as they do not vary over time. Conditioning on the sufficient statistic $\Sigma_t r_{it} \tilde{y}_{itg}$ not only gets rid of the α_i but also of the thresholds, even if they depend on i.

For each choice of g the Maximum Likelihood (ML) estimator, b_g say, yields consistent estimates for β if the model assumptions are satisfied. Analogous to the proof that the likelihood of the multinomial logit model is globally concave we can prove that the likelihood of a fixed effects binary choice logit model is globally concave. Therefore local optimization algorithms can be used to locate the global maximum.

In the second step, all the resulting estimates are combined and an Asymptotic Least Squares (ALS)[9] estimator is computed (using an estimate for the optimal weighting matrix). This step imposes the restrictions that b_g should not vary with g. Due to the linearity of these restrictions we can write down an explicit expression for the resulting ALS estimator b_{ALS}, $b_{ALS}=(A'WA)^{-1}A'Wb$, where A represents the linear restrictions,[10] W is the inverse of an estimator for the covariance matrix of b, where b consists of a vector of the first step estimates b_g. The asymptotic distribution of this estimator follows easily from standard ALS theory: $\sqrt{N}(b_{ALS} - \beta) \rightarrow N(0,(A'WA)^{-1})$

To compute the efficient ALS estimates we need to invert the covariance matrix of the first round estimates. The size of this matrix depends on the dimension of β and on the number of choices for g used. If the dimension of β is large we restrict the number of choices for g. We then only use g=5,6 and 7 in the first step of the procedure. We do not estimate $\gamma_2,..,\gamma_{R-1}$ in this procedure. However, this is not a serious disadvantage because we are interested in equivalence scales that can be computed from the estimated β_t's directly.

In the random effects model we additionally assume that the α_i are independent of $x_i=(x_{i1},..,x_{It})$ and that they are iid $N(0,\sigma_\alpha^2)$ distributed. The likelihood for an (un)balanced panel is then equal to

$$\prod_{i=1}^{N}\int_{-\infty}^{\infty}\left[\prod_{t=1}^{T}\{F_u(\gamma_{y_{it}+1}-\beta_t' x_{it}-\alpha)\ -\ F_u(\gamma_{y_{it}}-\beta_t' x_{it}-\alpha)\}^{r_{it}}\right] g(\alpha)\,d\alpha$$

where F_u is the distribution function of u_{it} and $g(\alpha)$ is the density of the α_i. In this model the thresholds γ_j are also estimated. The γ_j are not allowed to vary with i. We estimate the model using all the levels of the dependent variable.

In a pooled model we assume that $\alpha_i=0$ for all i and that the u_{it} are iid (standard) logistically distributed and that they are independent of $(x_1,..,x_T)$. Hence the observations in the pooled panel are iid over both i and t. Therefore, standard cross-section estimation procedures for an ordered response model can be applied to the pooled sample. We will apply Maximum Likelihood.

5. Estimation results

We performed the estimation procedure for the fixed effects panel data model described in section 4 for both satisfaction with life and satisfaction with income. The number of households in

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[9] For ALS see Kodde et al. (1990) and the references therein. ALS is just a special case of Minimum Distance estimation.

[10] $A=\iota_G \otimes I_{\dim(\beta)}$, where ι_G is a vector of ones of size G where G is the number of b_g's used in the first step, $\otimes$ is the Kronecker product and $I_{\dim(\beta)}$ is the identity matrix of size equal to the number of elements in β.

the sample, N, is equal to 4179 and T is equal to 8. The results are presented in table 3 and table 4 respectively. For satisfaction with life the regressors are chosen from the main explanatory variables described in the introduction and they include the logarithm of income, satisfaction with health, labour market state, age squared and the logarithm of household size. Due to the fixed effects approach, time constant regressors disappear into the fixed effect. This implies that, for example, parameters related to variables representing civil status and region of residence cannot be estimated. Although they can vary over time, these variables are constant over time for almost all households. We start by including income and household size only through log(income) and log(household size) because this specification is used in Van Praag and Van der Sar (1988) as well. The argument is that income per household equivalent matters, where household equivalent is household size to the power α. After taking the log, we obtain the specification used here (this interpretation disappears when including squared terms).

Because satisfaction with life covers all aspects of one's life, apart from log(income) and log(household size) we also include satisfaction with health, dummies for labour market state and age squared in estimation. Due to the high dimension of β_t and the large number of time periods we only use g=5,6 and 7 in estimation. This increases the number of households that do not contribute to the likelihood for all choices of g by 165, when compared to using al possible choices for g. Contrary to satisfaction with life, satisfaction with income relates to household income and hence it refers to the household situation and not only to the head of household's situation. Therefore we exclude variables that are specific for the head of household from the explanatory variables in the satisfaction with life equation. So, for satisfaction with income we only use log(income) and log(household size) as explanatory variables. The low dimension of β_t allows us to use g=0,..,9 in this model. Note that under the assumptions in section 2 initial assets in each period should be included. However, the value of initial assets is not available in our dataset. Therefore the effect of initial assets is assumed to be captured by the household specific effect and the remainder is assumed to be absorbed in the error term. Furthermore, prices are taken into account by using real income instead of nominal income.

Using satisfaction with life data, the variable y^*_{it} in (4) can be interpreted as the underlying satisfaction level on a continuous scale (i.e. the latent satisfaction level) which is transformed into a discrete variable to answer the satisfaction question. The coefficients can be interpreted in terms of the latent satisfaction level. We conclude that, in general, the coefficient related to log(household income) is significantly positive. Satisfaction with health has a significantly positive effect on satisfaction with life and the estimates are in the range [0.23,0.5]. An increase of one in satisfaction with health will lead to an increase between 0.23 and 0.5 in latent satisfaction with life.

Comparing this effect to the effect of income on latent satisfaction with life we conclude that for most years, *ceteris paribus*, income should be multiplied by numbers in the range 2 through 4 to establish the same increase in latent satisfaction with life as an increase of one in satisfaction with health. This demonstrates the importance of the effect of a change in satisfaction with health on satisfaction with life, compared to the effect of income changes. For the dummies related to labour market state (full-time employment is the reference level) the coefficient related to the unemployment dummy is significantly negative. This strong negative effect on satisfaction with life has been found before in Winkelmann and Winkelmann (1995). The nonparticipation dummy is significantly negative for 1984, 1985 and 1986. This might be due to the low social status associated with (voluntary) unemployment. Most time dummies are insignificant.

Because most parameter estimates related to log(household size) are (insignificantly) positive, an increase in the size of the household does not lead to a decrease in the satisfaction with life of the head of the household. Thus this increase in household size does not need to be compensated by an increase in income to maintain the level of satisfaction. Therefore, period-specific equivalence scales would even decrease with household size.

The objective function value in the ALS step can be used to perform an overidentifying restrictions test. The null hypothesis of no misspecification was rejected at any conventional significance level. This indicates that the model for satisfaction with life is misspecified and the results on period-specific equivalence scales can be due to model misspecification.

The results for satisfaction with income are presented in table 4. Log(household income) has a strong positive effect and the coefficient related to log(household size) is significantly negative, indicating that larger households are less satisfied with a given amount of income than smaller families. The effect of log(income) seems to be much stronger than the effect on satisfaction with life. However, the difference can be caused by the different set of explanatory variables used. If we estimate the model for satisfaction with life only including log(income) and log(household size), the effect of income is approximately one third of the effect of income on satisfaction with income and the coefficient of log(household size) is significantly negative for some, but not all, years. Again we conclude that income has a much stronger effect on satisfaction with income than on satisfaction with life.

In the results in table 4, income has a significantly positive effect and now the effect of an increase in household size is negative. This implies that an increase in household size has a negative effect on satisfaction with income that can be compensated by an increase in household income, to maintain the same satisfaction level. This differs from the results based on satisfaction with life. The period-specific equivalence scales for single person households, couples with one child, couples with two children and couples with four children are presented in table 5. Period-specific equivalence scales follow from equating latent satisfaction with income for a household with a given composition (i.e. the comparison household) to latent satisfaction of a reference household. The period-specific equivalence scale is equal to the income for the comparison household for which it reaches the same latent satisfaction level as the reference household, divided by the income of the reference household. The comparison household differs only in terms of income and household composition, so the remaining explanatory variables (if present) as well as α_i and the error term are assumed to be the same for both households. Due to the linear specification in log(real income) the logarithm of the ratio of the income of the comparison household and the reference household and hence the log of the period-specific equivalence scales can be computed easily. Taking exponentials yields the period-specific equivalence scales, which depend only on household composition and the parameter estimates. In the specification of table 3, household composition affects satisfaction with income only through household size so then the period-specific equivalence scales depend on household size and the parameter estimates. Using the Delta method and the covariance matrix of the parameter estimates, the standard errors are estimated. The estimates are fairly stable over time with values that are closest to one in 1987 and values farthest from one for 1991. A single person household needs approximately 0.7 times the expenditure of a couple to be as well off as the couple. This number is approximately 1.22 for couples with one child, 1.40 for a couple with two children and 1.7 for a couple with four children. All these numbers differ significantly from one. Several results on equivalence scales for the former West Germany are available in the literature. Merz and Faik (1995) use a 1983 cross-section on West Germany and they use a demand system approach to construct equivalence scales. They report values of 0.68, 1.17, 1.28 and 1.3, respectively. Especially the equivalence scale for a couple with four children is considerably lower than the number presented here. Van Praag et al. (1982) estimate poverty lines based on a West German sample of 1979. They use subjective data on the Income Evaluation Question (IEQ). The poverty lines lead to equivalence scale estimates of 0.83, 1.11, 1.20 and 1.34, respectively. We conclude that the equivalence scales based on the IEQ appear to be closer to one than the ones based on satisfaction with income. This finding is also reported in Melenberg and Van Soest (1996). Finally we note that the equivalence scales computed in this paper are smaller than those in Melenberg and Van Soest (1996) based on a 1984 cross-section of Dutch households. The difference between their and our results can stem from at least three sources. The first source is the use of a panel data model instead of a cross-section model. The second source is the different country analyzed and the third source is the different set of explanatory variables used.

Again the overidentifying restrictions test is carried out in the ALS step and again the null hypothesis of correct model specification is rejected. This result, together with the result on the satisfaction with life model indicates that we need a more general model. An important assumption in this ordered response model is the logit distributional assumption. Violation of this assumption leads to inconsistent estimates in this limited dependent variable type of model. Therefore it is worthwhile to consider semiparametric ordered response panel data models. To the best of our knowledge, the estimator proposed in Abrevaya (1996) is the only one available. Using semiparametric estimation techniques for the ORDP model is the topic of a subsequent paper, see Charlier (1997).

Another possible reason for misspecification of the previous models may be that ages of children were not taken into account when estimating the equivalence scales. To take ages of children into account, we use a specification that differs slightly from the ones in Kapteyn et al. (1988) and Melenberg and Van Soest (1996). We modify their specification to nest the model where the effect of household composition is only through log(household size). Let w_j be the weight given to household member j, where the household members are sorted in descending order on the basis of their age. We define $w_1=0$, $w_j=\ln(j/(j-1))$, $j>1$ and $hc=\Sigma w_j f(a_j)$ where hc stands for household composition, the summation is over all household members j, a_j is the age of household member j and

$$f(a_j) = \begin{cases} 1 & \text{if } a_j > 18 \\ 1+\pi_1(18-a_j)^2+\pi_2(18-a_j)^2(36+a_j) & \text{if } a_j \le 18 \end{cases}$$

Compared to Kapteyn et al. (1988) and Melenberg and Van Soest (1996) the modification we have made is that $w_1=0$ instead of 1, which implies that the oldest household member does not contribute. Then

$$hc = \log(hhsize)+\pi_1\Sigma_j w_j 1(a_j\le 18)(18-a_j)^2+\pi_2\Sigma_j w_j 1(a_j\le 18)(18-a_j)^2(36+a_j)$$

where 1(.) is an indicator function which is one if the condition between parentheses is satisfied and zero otherwise.

Now define $SUMWF1=\Sigma_j w_j 1(a_j\le 18)(18-a_j)^2/100$ and $SUMWF2=\Sigma_j w_j 1(a_j\le 18)(18-a_j)^2(36+a_j)/1000$. In estimation we included the additional terms

$$\pi_0 hc=\pi_0\log(hhsize) + 100\pi_0\pi_1 SUMWF1 + 1000\pi_0\pi_2 SUMWF2$$

and we allowed π_0, π_1 and π_2 to vary over time. In the tables following we will present estimation results for, among others, π_0, $100\pi_0\pi_1$ and $1000\pi_0\pi_2$, and not for π_1 and π_2. This should be kept in mind when testing or interpreting these coefficients from the tables. This model reduces to the previously estimated model if both π_1 and π_2 are zero.

The results in table 3 based on satisfaction with life indicate that an increase in household size need not be compensated by an increase in income. Based on the point estimates, equivalence scales would even decrease with household size using satisfaction with life. Therefore we will focus on the results using satisfaction with income in the remainder. Including the terms related to the ages of children and allowing the coefficients to vary over time leads to insignificant parameter estimates for (π_1,π_2), both for a given year and for all years simultaneously. Furthermore, the shape of the functions $f(a_j)$ varied tremendously for the years in the sample. Therefore it is more instructive to get an idea of the "average" effect of ages of children on the equivalence scale and we estimate a fixed effects model in which the parameters are not allowed to vary over time. This leads to the estimation results presented in columns two and three of table 6. As before all possibilities for g are used in estimation, so g=0,..,9. The coefficients related to log(income) and log(household size) are significant and they have the expected sign. Note that the estimates related to SUMWF1 and SUMWF2 are $100\pi_0\pi_1$ and $1000\pi_0\pi_2$, respectively, where π_0 is the coefficient related to LHHSIZE. Using the Delta method we conclude that the coefficients (π_1,π_2) are significant simultaneously. Compared to the results in table 4 the estimates for LINC and LHHSIZE are approximately equal to weighted averages of the estimates in table 4. Interpretation of the coefficients π_1 and π_2 can be done in a graphical way. These coefficients determine the function $f(a_j)$, where a_j is the age of household member j. A plot of the function $f(a_j)$ can be found in figure 2. The upper graph contains point estimates whereas the lower graph also contains pointwise 95% confidence bands.

Random effects estimates are presented in columns four and five of table 6. The effects of LINC and LHHSIZE are larger when compared to the fixed effect estimates in table 6. This might be due to positive correlation between LINC and the individual specific effect and negative correlation between LHHSIZE and the individual specific effect. The interpretation of this is that, *ceteris paribus*, households with high average income have a higher α_i and hence are more satisfied with their income than people with a low average income. Analogously, households with a large average size are less satisfied with their income. Furthermore, the standard errors in the random effects model are lower than for the fixed effects model. A plot of the function $f(a_j)$ based on the random effects estimates can also be found in figure 2. The upper graph contains point estimates whereas the lower graph again contains pointwise 95% confidence bands. Compared to the fixed effects estimates, children contribute more to household composition, see the definition of hc earlier in this section.

Finally, we present the pooled estimates in columns six and seven of table 6. The parameters related to LINC and LHHSIZE are significant and they are in between the fixed and random effects estimates. The parameters π_1 and π_2 are significant now, both individually and simultaneously. A plot of the function $f(a_j)$ can be found in figure 2 and different from the fixed or random effects estimates, the function is decreasing for low ages whereas it is increasing after the age of six. A comparable pattern was found in Muffels et al. (1990), based on Dutch data.

Period-specific equivalence scales for the fixed and random effects model and the pooled model are presented in table 7. They now also depend on the ages of children, if present, due to the inclusion of SUMWF1 and SUMWF2 in household composition. The estimates based on the random effects model are further away from one than the estimates based on the fixed effects model. Due to the smaller standard errors of the parameters, the period-specific equivalence scales are estimated more precisely for the random effects model. The estimates for the period-specific equivalence scales based on the fixed effects model are closest to one. The estimates based on the pooled model are in between the estimates based on the fixed and the random effects model.

Results for the (*ex post*) lifetime equivalence scales based on ρ=r are presented in table 8. In computation we will follow Banks et al. (1994a) and assume that children leave the household at age 18 and that the lifetime is the period between 20 and 60 years of age. Hence C in the expression for lifetime equivalence scales is equal to 39. The lifetime equivalence scales are the average of the period-specific equivalence scales over the lifetime. Because the period-specific equivalence scales depend on the parameter estimates, the lifetime equivalence scales also depend on the parameter estimates. Therefore, standard error estimates can be obtained using the Delta method.

The results based on the fixed effects estimates are again closer to one than the results using the random effects or the pooled estimates. Because ρ is equal to r, the estimates do not depend on the age of the head of household at the times children are born. The only aspect that matters is the time between the births. Based on the fixed effects estimates, a couple having one child needs to spend approximately 1.07 times as much over the life-cycle as a couple without a child. If children are born two years after the previous child, lifetime equivalence scales for two children and three children are 1.125 and 1.174, respectively. All this is computed under the assumption that children leave the household at age 18 and that the life-cycle is defined over a period of 40 years.

Comparing the results to the results in Banks et al. (1994a) we have to take into account that their reference household is the same but they normalize the related equivalence scale to 2 instead of 1. Renormalizing to 1, their results lead to life-time scales of 1.08, 1.2 and 1.38, respectively. The first one is close to the one presented in table 8 whereas the latter two are much smaller for most models in table 8. However, as noted before, the results by Banks et al. (1994a) are arbitrary in the sense that demand data alone cannot identify the equivalence scales.

Finally we tested for dependence between the household specific effects α_i and the explanatory variables. We performed a Hausman-type test comparing the fixed effects and the random effects estimates for the parameters related to LINC, LHHSIZE, SUMWF1 and SUMWF2. Under the null hypothesis of no dependence both estimators are consistent whereas under the alternative the fixed effects estimator is still consistent and the random effects estimator is not. The test can be performed easily when writing the fixed effects and random effects estimators, b_{FE} and b_{RE}, say, in terms of influence functions. Using these influence functions it is easy to construct a consistent positive definite estimator for the covariance matrix of the difference between the two estimators. This then is used to perform a χ^2 test. The degrees of freedom are equal to the number of parameter estimates compared, i.e. 4 in our case. The test statistic is equal to 177.8 which is much larger than the critical value of the χ^2_4 distribution at any conventional significance level. The result is mainly due to the precise but different estimates related to LINC and LHHSIZE (see table 6). Because the pooled model is a restricted version of the random effects model, the pooled specification will be rejected as well, when compared to the fixed effects model. Thus the random effects and pooled model are misspecified. Using an overidentifying restrictions test we also conclude that even this fixed effects panel data model is misspecified, so an even more general model might be preferred.

An other reason for misspecification could be the exclusion of log(income) squared, log(household size) squared and their cross-product, for example. Including these terms still led to the conclusion that, on the basis of the overidentifying restrictions test, the resulting model is misspecified. Besides, including these terms would lead to equivalence scales that depend on income, which is not feasible from a policy point of view. Therefore we restrict attention to a specification including log(income) and log(household size).

6. Conclusions

In this paper we have tried to answer questions like how much a household with four children needs to spend compared to a household with two children or how much a childless couple needs to spend compared to a single person household to attain the same welfare level. The answers to these questions are important because for example poverty thresholds, child allowances and social benefits are based on them and in an intertemporal setting they can be used to assess the cost of children over the life-cycle. Especially for the latter, we have discussed the definition of equivalence scales in an intertemporal setting under uncertainty. We conclude that it is possible to estimate *ex post* lifetime equivalence scales. Estimation of *ex ante* equivalence scales is not possible. We provided answers for West Germany based on the GSOEP.

Using satisfaction with life data we do not find a decrease in satisfaction if household size increases. This implies that on the basis of these data an increase in household size does not need to be compensated. Testing the specification, however, indicated that the model is misspecified so conclusions from the results might not be valid due to model misspecification.

Using satisfaction with income data we do find a decrease in satisfaction if household size increases. This implies that on the basis of these data larger households should be compensated. These period-specific equivalence scales are used to estimate lifetime equivalence scales. On the basis of the most general model estimated in this paper we find period-specific equivalence scales that are approximately 1.12 for a couple with a six-year-old child, 1.2 for a couple with a twelve-year-old child and 1.3 for a couple with a six-year-old child and a twelve-year-old child. Using these period-specific equivalence scales, lifetime equivalence scales are constructed. The lifetime equivalence scales indicate that a couple having one child needs to spend approximately 1.07 times as much over the life-cycle as a couple without a child, to reach the same lifetime welfare as the reference household. If children are born two years after the previous child, lifetime equivalence scales for a couple with two or three children are 1.125 and 1.174, respectively.

Specification testing indicates that all three models are misspecified. This might be due to the distributional assumptions in the models. Using semiparametric estimation techniques for an ORDP model is the topic of a subsequent paper, see Charlier (1997).

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Figure 1: *Nonparametric estimates for the relation between satisfaction with life and log(income), satisfaction with income and log(income), satisfaction with life and household size and satisfaction with income and household size*

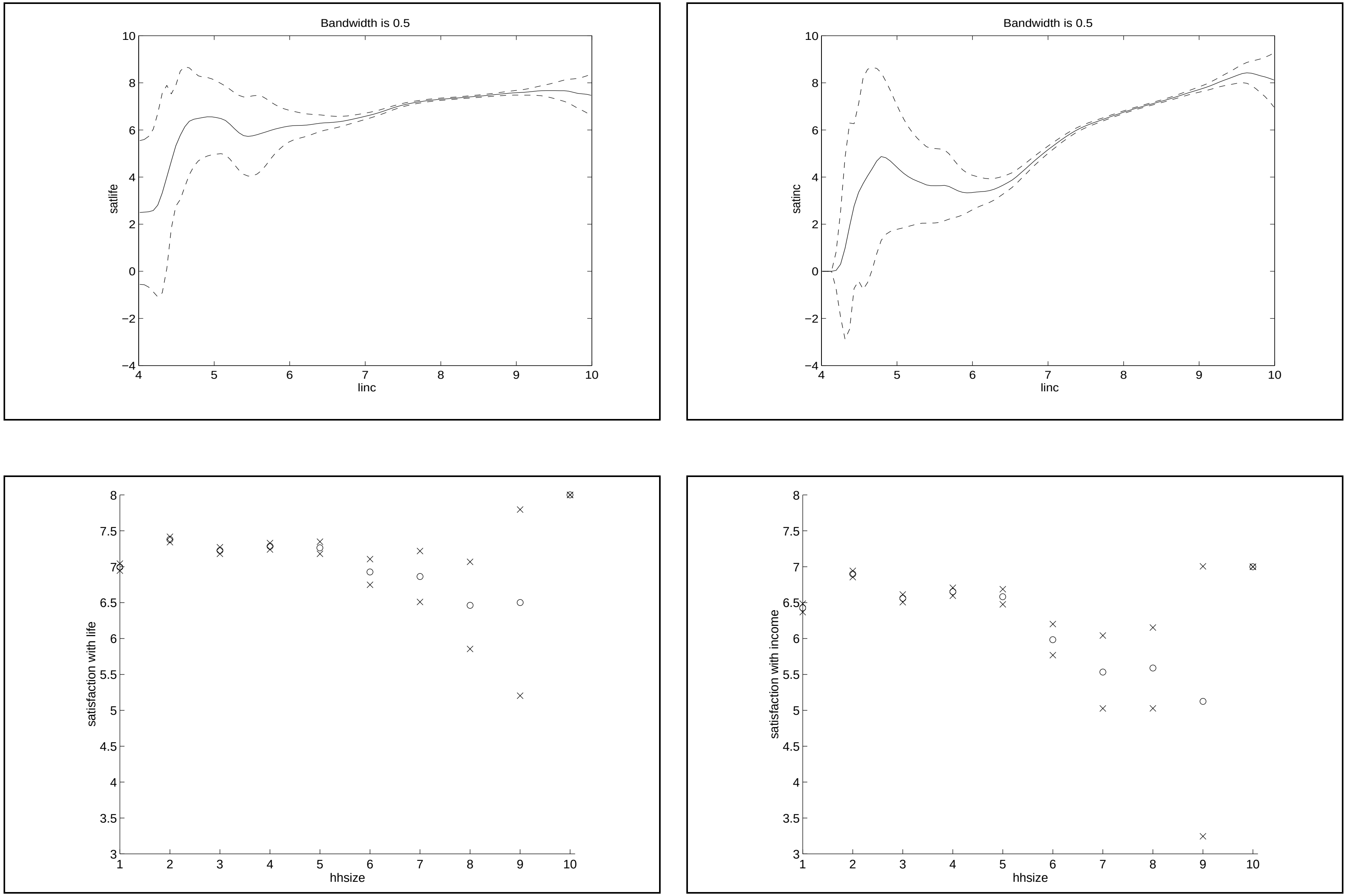
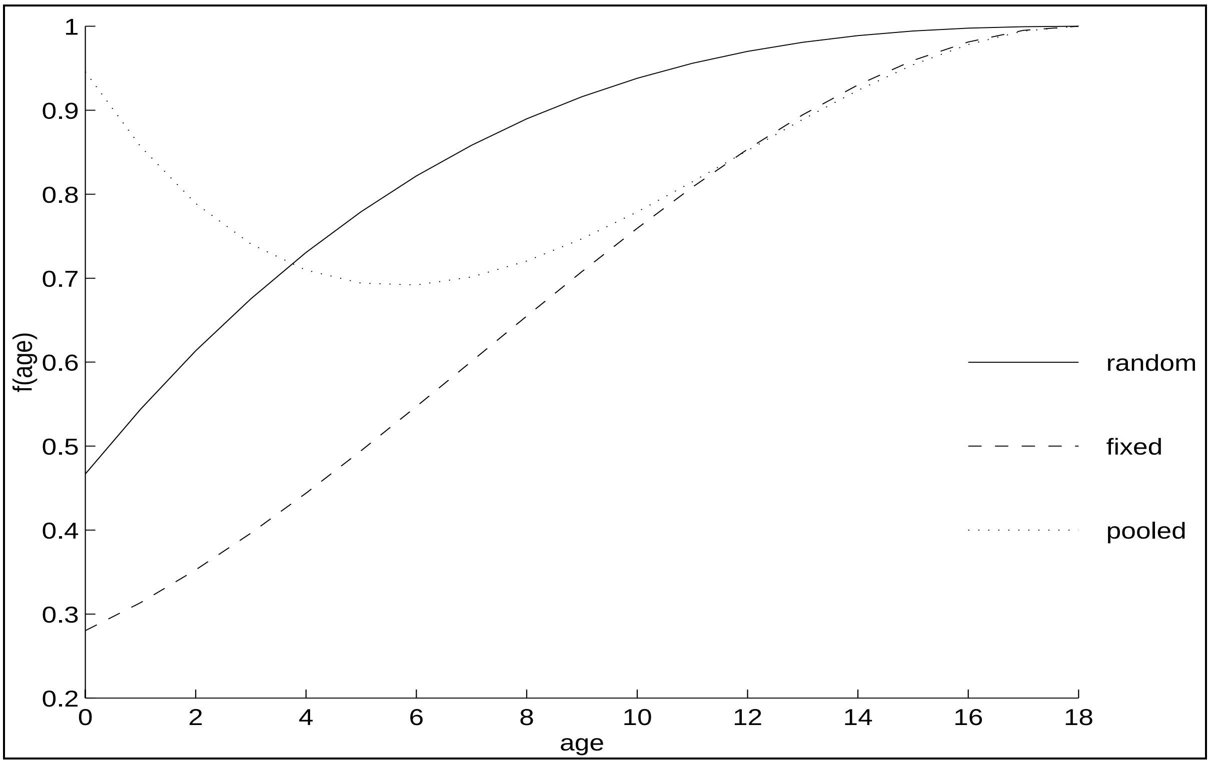


Figure 2: *Graphs of the function f(a), defined in section 5. The upper graph contains point estimates for the function f(a) for different estimators; the lower graph contains pointwise 95% confidence bands as well.*



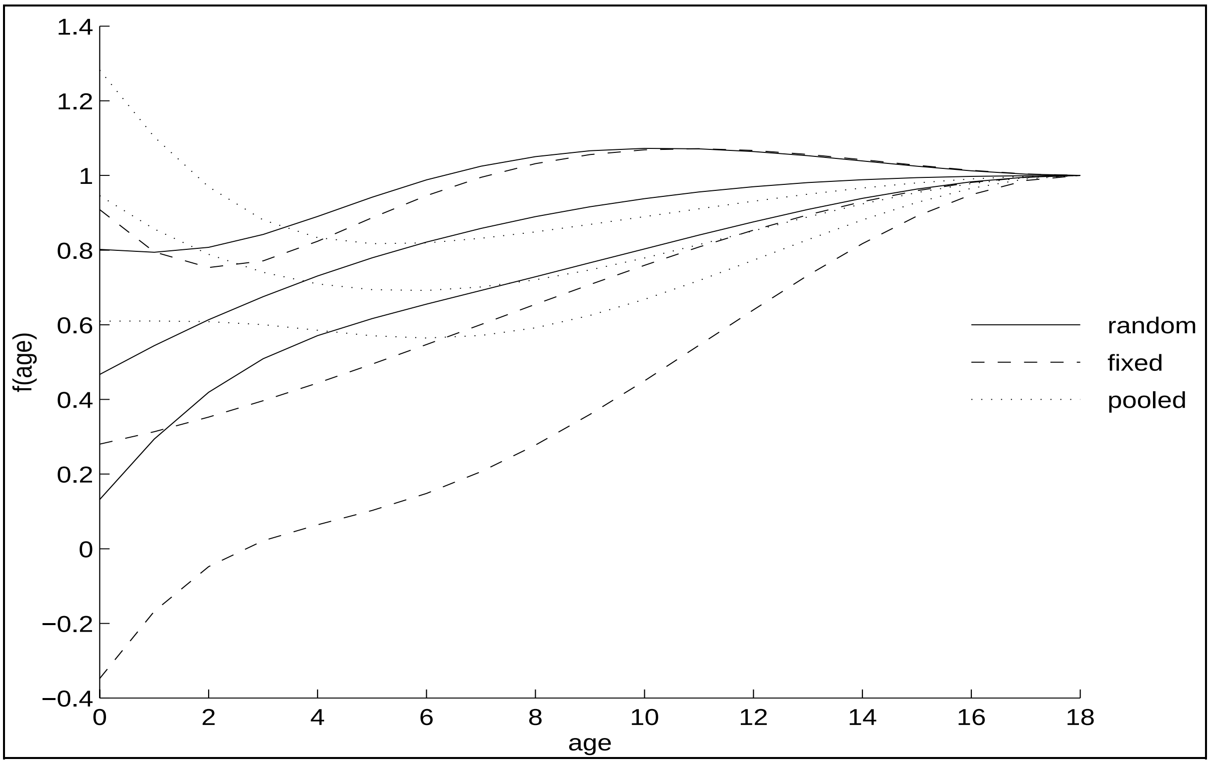


Table 1: Definition and overview of variables

Variable	Definition
SATLIFE	satisfaction with life of head of household, measured on a scale from 0 (very dissatisfied) to 10 (very satisfied)
SATINC	satisfaction with household income responded by the head of household, measured on a scale from 0 (very dissatisfied) to 10 (very satisfied)
LINC	log(real total household income) in Deutschmark/month
LHHSIZE	log(household size)
SATHLTH	satisfaction with health of head of household, measured on a scale from 0 (very dissatisfied) to 10 (very satisfied)
FULLEMP	dummy equal to one for a full-time employed head of household, zero otherwise
PARTEMP	dummy equal to one for a part-time employed head of household, zero otherwise
UNEMPL	dummy equal to one for an (involuntarily) unemployed head of household, zero otherwise
NONPART	dummy equal to one for a nonparticipating head of household, zero otherwise
SELFEMP	dummy equal to one for a self-employed head of household, zero otherwise
AGE	age of the head of household divided by 10
AGE2	AGE squared
DUMMY85-DUMMY91	time dummies for respectively 1985 till 1991

Table 2: Summary statistics for the variables described in table 1 (standard deviation in parentheses)

	1984	1985	1986	1987	1988	1989	1990	1991
# obs	3926	3470	3187	3041	2827	2634	2509	2343
SATLIFE	7.40	7.22	7.34	7.14	7.00	7.07	7.28	7.36
	(2.11)	(2.00)	(1.86)	(1.88)	(1.91)	(1.92)	(1.75)	(1.70)
SATINC	6.34	6.42	6.59	6.58	6.60	6.73	6.81	7.01
	(2.59)	(2.44)	(2.26)	(2.26)	(2.21)	(2.14)	(2.10)	(1.99)
LINC	7.84	7.87	7.94	7.97	8.00	8.03	8.06	8.08
	(0.58)	(0.56)	(0.54)	(0.54)	(0.53)	(0.53)	(0.53)	(0.52)
LHHSIZE	0.78	0.80	0.81	0.81	0.82	0.82	0.82	0.82
	(0.54)	(0.54)	(0.53)	(0.53)	(0.53)	(0.53)	(0.53)	(0.52)
SATHLTH	6.67	6.65	6.63	6.56	6.37	6.35	6.34	6.37
	(2.69)	(2.47)	(2.42)	(2.35)	(2.40)	(2.41)	(2.31)	(2.28)
FULLEMP	0.62	0.62	0.62	0.62	0.62	0.61	0.60	0.60
PARTEMP	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.03
UNEMPL	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.01
NONPART	0.30	0.30	0.30	0.30	0.31	0.32	0.34	0.33
SELFEMP	0.06	0.06	0.06	0.07	0.07	0.06	0.06	0.06
AGE	4.92	4.97	5.03	5.09	5.18	5.23	5.31	5.37
	(1.66)	(1.62)	(1.60)	(1.58)	(1.55)	(1.54)	(1.53)	(1.49)

Table 3: Results based on a fixed effects model for satisfaction with life (standard errors in parentheses)

parameter	1984	1985	1986	1987	1988	1989	1990	1991
LINC	0.330^{**}	0.372^{**}	0.078	0.528^{**}	0.234^{*}	0.284^{**}	0.370^{**}	0.374^{**}
	(0.099)	(0.109)	(0.115)	(0.110)	(0.116)	(0.113)	(0.124)	(0.121)
LHHSIZE	0.390^{**}	0.043	0.141	−0.050	0.168	-0.277^{*}	−0.048	0.146
	(0.114)	(0.118)	(0.126)	(0.122)	(0.128)	(0.125)	(0.130)	(0.124)
SATHLTH	0.232^{**}	0.304^{**}	0.412^{**}	0.320^{**}	0.329^{**}	0.260^{**}	0.283^{**}	0.503^{**}
	(0.018)	(0.021)	(0.022)	(0.021)	(0.022)	(0.021)	(0.023)	(0.023)
PARTEMP	-0.861^{**}	−0.454	−0.346	0.015	-0.944^{**}	−0.030	0.232	0.493
	(0.216)	(0.241)	(0.270)	(0.251)	(0.361)	(0.267)	(0.280)	(0.264)
UNEMPL	-1.194^{**}	-0.817^{**}	-1.292^{**}	-0.816^{**}	-0.538^{*}	−0.535	-0.867^{**}	-1.353^{**}
	(0.208)	(0.216)	(0.232)	(0.232)	(0.237)	(0.302)	(0.308)	(0.343)
NONPART	-0.408^{**}	-0.620^{**}	-0.444^{**}	−0.058	−0.086	−0.015	0.144	0.037
	(0.158)	(0.167)	(0.176)	(0.177)	(0.183)	(0.168)	(0.181)	(0.172)
SELFEMP	−0.102	-0.395^{*}	−0.387	−0.312	-0.570^{**}	−0.344	-0.559^{**}	−0.171
	(0.179)	(0.197)	(0.205)	(0.199)	(0.210)	(0.197)	(0.204)	(0.193)
AGE2	0.564^{**}	0.559^{**}	0.552^{**}	0.533^{**}	0.523^{**}	0.499^{**}	0.484^{**}	0.488^{**}
	(0.157)	(0.154)	(0.151)	(0.148)	(0.145)	(0.142)	(0.140)	(0.137)
TIME DUMMY		−1.079	0.136	-3.105^{**}	−1.535	−0.970	−1.248	-3.368^{**}
		(1.021)	(1.060)	(1.018)	(1.108)	(1.097)	(1.178)	(1.189)

* means significant at the 5% level; ** means significant at the 1% level

Table 4: *Results based on a fixed effects model for satisfaction with income (standard errors in parentheses)*

year	parameter	estimates	
1984	LINC	1.778**	(0.072)
	LHHSIZE	−0.785**	(0.092)
1985	LINC	1.598**	(0.081)
	LHHSIZE	−0.794**	(0.097)
1986	LINC	1.537**	(0.088)
	LHHSIZE	−0.786**	(0.099)
1987	LINC	1.371**	(0.088)
	LHHSIZE	−0.535**	(0.102)
1988	LINC	1.301**	(0.090)
	LHHSIZE	−0.584**	(0.103)
1989	LINC	1.292**	(0.090)
	LHHSIZE	−0.570**	(0.105)
1990	LINC	1.138**	(0.094)
	LHHSIZE	−0.619**	(0.104)
1991	LINC	1.092**	(0.097)
	LHHSIZE	−0.628**	(0.107)
	DUMMY85	1.485*	(0.714)
	DUMMY86	1.977**	(0.750)
	DUMMY87	3.020**	(0.768)
	DUMMY88	3.601**	(0.763)
	DUMMY89	3.765**	(0.795)
	DUMMY90	5.082**	(0.814)
	DUMMY91	5.710**	(0.821)

* means significant at the 5% level; ** means significant at the 1% level

Table 5: *Period-specific equivalence scales based on satisfaction with income (household size equal to 2 is the reference household, standard errors in parentheses)*

Year	HS=1		HS=3		HS=4		HS=6	
1984	0.736	(0.024)	1.196	(0.023)	1.358	(0.044)	1.624	(0.083)
1985	0.708	(0.026)	1.223	(0.027)	1.412	(0.053)	1.727	(0.102)
1986	0.701	(0.028)	1.230	(0.028)	1.426	(0.056)	1.754	(0.110)
1987	0.763	(0.035)	1.171	(0.032)	1.311	(0.061)	1.535	(0.113)
1988	0.732	(0.036)	1.200	(0.035)	1.365	(0.067)	1.638	(0.128)
1989	0.736	(0.037)	1.196	(0.035)	1.358	(0.068)	1.624	(0.129)
1990	0.686	(0.038)	1.247	(0.041)	1.458	(0.082)	1.818	(0.161)
1991	0.671	(0.040)	1.262	(0.044)	1.489	(0.089)	1.880	(0.178)

HS=Household Size, HS=2 is the reference case.

Table 6: *Estimation results based on satisfaction with income, taking ages of children into account (standard errors in parentheses)*

parameter	fixed effects		random effects		pooled	
LINC	1.491	(0.048)**	1.923**	(0.030)	1.685**	(0.027)
LHHSIZE	−0.769	(0.080)**	−1.121**	(0.040)	−0.990**	(0.030)
SUMWF1	−0.255	(0.860)	0.458	(0.595)	−1.155*	(0.498)
SUMWF2	0.118	(0.227)	−0.076	(0.154)	0.325**	(0.126)
DUMMY84	0.0		−8.527**	(0.219)	−8.917**	(0.203)
DUMMY85	0.047	(0.047)	−8.470**	(0.220)	−8.893**	(0.204)
DUMMY86	0.100	(0.047)*	−8.460**	(0.222)	−8.891**	(0.206)
DUMMY87	−0.001	(0.049)	−8.539**	(0.223)	−8.949**	(0.207)
DUMMY88	0.004	(0.049)	−8.565**	(0.224)	−8.973**	(0.208)
DUMMY89	0.087	(0.051)	−8.490**	(0.226)	−8.934**	(0.208)
DUMMY90	0.139	(0.051)*	−8.475**	(0.225)	−8.925**	(0.209)
DUMMY91	0.381	(0.052)**	−8.289**	(0.228)	−8.793**	(0.210)
σ_α^2			2.183	(0.092)	0.0	
γ_1			0.5		0.5	
γ_2			1.033	(0.030)	0.972	(0.029)
γ_3			1.832	(0.038)	1.652	(0.039)
γ_4			2.705	(0.043)	2.360	(0.044)
γ_5			3.386	(0.044)	2.890	(0.045)
γ_6			4.720	(0.046)	3.892	(0.047)
γ_7			5.450	(0.047)	4.422	(0.048)
γ_8			6.532	(0.048)	5.197	(0.049)
γ_9			8.122	(0.051)	6.358	(0.050)
γ_{10}			9.098	(0.053)	7.105	(0.053)

* means significant at the 5% level; ** means significant at the 1% level
Normalization: $\sigma_u^2=\pi^2/3$

Table 7: Period-specific equivalence scales based on satisfaction with income (household size equal to two adults is the reference household, standard errors in parentheses)

NA[a]	ages of children				fixed effects	random effects	pooled
1					0.699	0.668	0.665
					(0.024)	(0.008)	(0.007)
2					1.000	1.000	1.000
2	6				1.121	1.214	1.179
					(0.046)	(0.022)	(0.017)
2	12				1.195	1.258	1.225
					(0.030)	(0.014)	(0.011)
2	12	6			1.297	1.443	1.377
					(0.068)	(0.033)	(0.025)
2	18	12	6	1	1.535	1.757	1.760
					(0.084)	(0.040)	(0.031)

[a] NA is Number of Adults

Table 8: Lifetime equivalence scales based on $\rho=r$ (lifetime ranges from age 20 of the head of household to age 60, standard errors in parentheses)

NA[a]	age of head household at times of birth			fixed effects	random effects	pooled
2				1.000	1.000	1.000
2	26			1.068	1.099	1.097
				(0.013)	(0.006)	(0.005)
2	26	28		1.125	1.186	1.181
				(0.024)	(0.012)	(0.009)
2	26	28	30	1.174	1.266	1.258
				(0.031)	(0.017)	(0.012)

[a] NA is Number of Adults

Appendix A

In this appendix we present the questions on satisfaction with life/satisfaction with income that were answered by the respondents as well as the monthly net household income question.

1. *How satisfied are you today with the following areas of your life? Please answer by using the following scale, in which 0 means totally unhappy, and 10 means totally happy. If you are partly happy and partly not, select a number in between. How satisfied are you...*

   [AP0302] with your household income?

2. *At the end we like to ask you for your satisfaction with your entire life. Please answer by using the following scale, in which 0 means totally unhappy, and 10 means totally happy.*

   [AP6801] How happy are you at present with your life as a whole?

3. *[AH46] If everything is taken together: how high is the total monthly income of all the household members at present? Please give the monthly net amount, the amount after the deduction of tax and national insurance contributions. Regular payments such as rent subsidy, child benefit, government grants, subsistence allowances, etc., should be included. If not known exactly, please estimate the monthly amount.*

   DM per month _____